4

Hydraulics of Fuel Injection

Efficient combustion requires that the fuel be injected at the proper time and rate, and the injection pressure must be sufficiently high for adequate atomization and penetration. This involves not only the mechanical characteristics of the pump, discharge tubing, and nozzle of the conventional jerk pump system, but also the compressibility and dynamics of the fuel column between the pump and nozzle.

FUEL COMPRESSIBILITY

Since fuel is compressible there is a time lag between the beginning of delivery by the pump and the beginning of discharge from the nozzle, and the rate of delivery from the pump is not identical with the rate of discharge from the nozzle.

The compressibility of a liquid is the reciprocal of the modulus of elasticity or bulk modulus, as it is also known. The modulus of elasticity can be expressed the same for liquids as it is for solids:

$$K = \frac{\text{change of stress}}{\text{change of strain}} = \frac{\text{increase of pressure}}{\text{decrease of specific volume}}$$

where K = modulus of elasticity.

Thus, if the initial volume V_o and pressure p_o is changed to V_1 and p_1, respectively,

$$K = \frac{p_1 - p_o}{\dfrac{V_o - V_1}{V_o}} = \frac{V_o}{V_o - V_1}(p_1 - p_o)$$

Dow and Fink[1] found that, within the range of experimental error, all oils have relatively the same compressibility. They correlated considerable data on the compressibility of oils and developed the empirical equation,

$$\rho = \rho_o(1 + ap - bp^2)_t$$

Where:

- **ρ** = density in g/cc at temperature t (F°) and pressure p (psi)
- **ρ_0** = density in g/cc at same temperature and atmospheric pressure.
- **a,b** = constants at any given temperature.

Using this equation a family of curves of mean modulus of elasticity vs. pressure was plotted as shown in Fig. 52. Likewise, a family of curves of mean modulus of elasticity vs. temperature at constant pressure was plotted as shown in Fig. 53. The mean modulus used herein is the average modulus over the pressure range from zero to the pressure under consideration. For the majority of fuel injection applications the mean modulus of elasticity varies from 230,000 to 260,000 psi depending upon the maximum injection pressure and operating temperature.

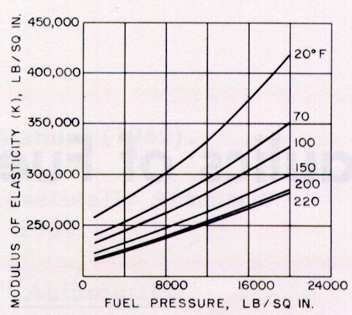


Fig. 52. Effect of pressure on modulus of elasticity of petroleum oils.

Fuel Storage at Pressure

By solving for V in equation 6, the volume of fuel V that can be stored at a pressure pl in a reservoir of volume v_o is,

$$V = \frac{V_o(p_1 - p_o)}{(K + p_o - p_1)}$$

PRESSURE WAVES IN FUEL LINES

As fuel oil is compressible a pressure wave initiated by pump plunger movement is propagated through the discharge tubing at the speed of sound in the fuel oil. The pressure wave is caused by the pressure built up by the plunger in compressing the fuel at the pump, while accelerating the fuel column towards the nozzle. These waves not only travel from the pump to the nozzle, but they are also reflected back towards the pump from the nozzle. The pressure waves alter the designed rates of injection and pressures, so that theoretical and experimental methods of analysis and control have been developed.

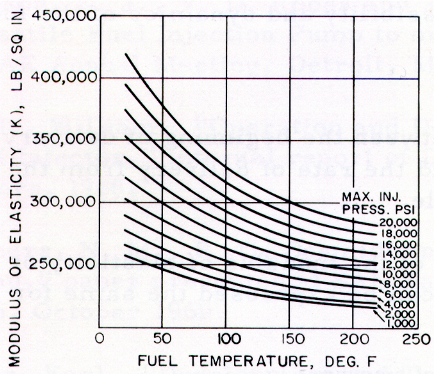


Fig. 53. Effect of temperature on modulus of elasticity of petroleum oils.

Velocity of Pressure Waves

Any disturbance, such as a change in pressure at a point in the system, is propagated to all points in the fluid with the speed of sound. Thus, the acoustic velocity is,

$$v_s = \sqrt{\frac{Kg}{\rho}} \quad \text{ips}$$

where:

- **K** = modulus of elasticity, psi
- **g** = 386 ips^2
- **ρ** = density of fluid, lb/cu. in.

An average fuel oil with a specific gravity of 0.85 has a density of 0.0308 lb/cu.in. Assuming K = 260,OO.O psi,

$$V_s = \sqrt{\frac{260,000 \times 386}{.0308}} = \frac{57200}{12} = 4760 \text{ fps}$$

Amplitude of Pressure Waves

A change in the velocity of the pressure wave produces a corresponding change in the pressure, and this change in pressure is propagated with the speed of sound in the fluid. Thus, the higher the velocity of the fuel entering the discharge tubing, the greater will be the piling up of the fuel as a result of its inertia and compressibility. Consequently, the higher will be the pressure of the wave propagated from that point. The relationship for this change is,

$$V_s = \sqrt{\frac{260,000 \times 386}{.0308}} = \frac{57200}{12} = 4760 \text{ fps} \qquad (10)$$

Accordingly, the change in pressure is directly proportional to the change in velocity:

$$\Delta p = \Delta v \sqrt{\frac{K\rho}{g}} \qquad (11)$$

where:

- **Δp** = Change in pressure, psi
- **Δv** = Change in velocity, ips

For example, the initial pressure developed in the discharge tubing when the barrel ports are covered by plunger on its delivery stroke is,

$$p = V_t \sqrt{\frac{K\rho}{g}} = v_p \frac{A_{pL}}{A_t} \sqrt{\frac{K\rho}{g}} = v_p \left(\frac{D}{d}\right)^2 \sqrt{\frac{K\rho}{g}}$$
$$= v_p \left(\frac{D}{d}\right)^2 \sqrt{\frac{260,000 \times 0.0308}{386}} = 4.55 \left(\frac{D}{d}\right)^2 v_p \text{, psi} \qquad (12)$$

where:

- $\mathbf{V_p}$ = plunger velocity, ips
- **D** = plunger dia. in.
- **d** = tubing bore, in.

Examination of this last equation reveals that three methods for increasing the amplitude of the pressure wave are: (1) Larger plunger diameter, (2) smaller tubing bore, and (3) higher plunger velocity. This equation does not account for friction which limits the minimum usable tubing bore.

Effect of Tubing Expansion

Fuel discharge tubings have such heavy walls that the effect of their elasticity on fuel injection is insignificant. The increase in the tubing bore radius with pressure can be represented by the equation,[2]

$$\Delta r = p \frac{r}{E}\left(\frac{R^2 + r^2}{R^2 - r^2} + \lambda\right) \tag{13}$$

where:

- **r** = radius tubing bore, in.
- **R** = outer radius of tubing, in.
- **r** = change in inner radius, in.
- **λ** = Poisson's ratio = 0.26 for steel

The outside diameter of most discharge tubings is at least twice the bore. For a tubing of this proportion and peak injection pressure of 15,000 psi, the resultant expansion is:

$$\Delta d = \frac{pr}{E}\left(\frac{4r^2 + r^2}{4r^2 - r^2} + .26\right) = 1.93\frac{dp}{E} = \frac{1.93d \times 15000}{30{,}000{,}000} = 0.00097d$$

This corresponds to an increase of bore area of only 0,2%, which is negligible.

Dr. Sass[3] has shown that the decrease in velocity of the pressure wave from expansion of the tubing is given by,

$$\Delta v_s^1 = \sqrt{\frac{Ed}{Ed + KD}}\sqrt{\frac{Kg}{\rho}} \tag{14}$$

where:

- **D** = outside diameter of tubing
- **d** = inside diameter of tubing.

For D = 2d, K = 260,000 psi, E = 30,000,000 psi, and as

$$v_s = \sqrt{\frac{Kg}{\rho}}$$ the reduction in velocity is:

$$\Delta v_s^1 = v_s\sqrt{\frac{E}{E + 2K}} = v_s\sqrt{\frac{30{,}000{,}000}{30{,}000{,}000 + 2 \times 260{,}000}} = 0.992\ v_s$$

Since the reduction in velocity is less than one percent, the effect of tubing elasticity can be disregarded.

ANALYSIS

From the introduction of solid injection in 1910 until Dr. Sass published his work[3] in 1929 based on Allievi's theory of water hammer, very little was known about the injection process. Since then various theoretical methods have been devised and, although bench and engine tests are still relied upon in selecting and developing injection system components, they can be helpful in predetermining or evaluating the effects of various changes. Unfortunately, these theoretical methods are quite tedious unless computer facilities are available, and the accuracy of the results is dependent upon the simplifying assumptions made. As a study of some of these methods may be helpful in understanding the hydraulics of fuel injection, three different methods of analysis are presented.

MATHEMATICAL METHOD NEGLECTING PRESSURE WAVES[4]

This method, which considers fuel compressibility, is applicable only when the pressure wave energy is small, such as in unit injectors, or at low pump speeds and low rates of pump delivery. The following factors were considered as affecting the instantaneous pressures p built up at the discharge orifice of the injection system:

- $\mathbf{v_\theta}$ = velocity of pump plunger at angle, θ, inch/deg
- $\mathbf{A_p}$ = area of pump plunger, sq. in.
- $\mathbf{A_0}$ = total orifice area, sq. in.
- $\mathbf{A_t}$ = area of tubing bore, sq. in.
- $\mathbf{C_d}$ = nozzle coefficient of discharge
- **L** = length of discharge tubing and other passages, in.
- **n** = pump speed, rpm
- $\mathbf{P_a}$ = nozzle opening pressure, psi
- $\mathbf{P_r}$ = residual pressure in discharge tubing, psi
- $\mathbf{V_\theta}$ = total fuel volume between plunger and nozzle orifices at angle θ, cu. in.

Maximum Injection Pressure

Considering only pump and nozzle factors, the maximum pressure pm is obtained by equating the rate of plunger displacement to the rate of discharge through the nozzle orifice. Thus,

Plunger displacement rate
$$Q_p = A_p v_\theta \times \frac{n\ 360}{60} = 6nA_p v_\theta \text{ cu. in./sec} \tag{15}$$

Maximum nozzle discharge rate,
$$Q_N = C_d A_o \sqrt{2g \frac{P_m}{\rho}} \tag{16}$$

Equating these two rates and solving for Pm'

$$p_m = \frac{18\,(A_p\, n\, v_\theta)^2 \rho}{(A_o C_d)^2 g} \tag{17}$$

Accordingly, the maximum pressure varies directly as the square of the pump speed and inversely as the square of the nozzle orifice area. The rate of injection then varies directly with the speed. The maximum pressures are never realized, however, because of compressibility and inertia of the fuel and resistance to flow.

Instantaneous Injection Pressure Considering Compressibility

When the pump plunger starts its effective stroke, fuel does not issue from the nozzle, particularly with spring loaded nozzles, until the fuel pressure developed exceeds the nozzle opening pressure. Thereafter, assuming instantaneous pressure transmittal and negligible resistance to flow through the discharge tubing, the plunger displacement rate is equal to the sum of the rates of nozzle discharge and fuel compression. The rate of fuel compression is the product of rate of pressure change per second, coefficient of compressibility, and volume of fuel acted upon.

Therefore,

$$6nA_p v_\theta = A_o C_d \sqrt{\frac{2gp}{\rho}} + 6n \frac{dp}{d\theta} \frac{V_\theta}{K} \tag{18}$$

Since V_θ is a function of pump angle, an approximate solution of this equation can be made by substituting

$\frac{p_b - p_a}{\theta_b - \theta_a}$ for $\frac{dp}{d\theta}$. Thus,

$$6nA_pV_\theta = A_oC_d\sqrt{\frac{2gp_b}{\rho}} + \frac{6n\,(p_b - p_a)}{(\theta_b - \theta_a)}\frac{R_\theta}{K} \tag{19}$$

Angles θ_b, and θ_a, are taken close together so that the error of approximation is small, and the initial value of P_a is taken as the nozzle opening pressure p_o. By solving Eq. (15) as a quadratic in P_b,

$$\sqrt{p_b} = \frac{-A_oC_d\sqrt{\frac{2g}{\rho}} + \sqrt{\frac{2A_o^2C_d^2\,g}{\rho} + \frac{144\,n^2A_p\,v_\theta V_\theta}{(\theta_b - \theta_a)\,K} + \frac{144\,n^2\,V_\theta^2}{(\theta_b - \theta_a)^2K^2}\,p_a}}{\frac{12n}{(\theta_b - \theta_a)}\frac{V_\theta}{K}} \tag{20}$$

This equation gives the value of the injection pressure P_b at any point b up to the time of fuel cut off by the pump. Then if the delivery valve closes instantaneously, the compressed fuel trapped in the discharge tubing will expand and continue to discharge through the nozzle until its closing pressure is reached. Therefore, the rate of nozzle discharge after the end of pump delivery is equal to the rate of expansion of the fuel in the injection tube. Accordingly,

$$A_oC_d\sqrt{\frac{2gp}{\rho}} = -\frac{V_2}{K}\frac{dp}{dt} \tag{21}$$

$$\text{Integrating,}\quad t = \frac{V_2}{KA_oC_d}\sqrt{\frac{2\rho}{g}}\left(\sqrt{p_3} - \sqrt{p}\right) \tag{22}$$

Solving for the instantaneous pressure p at t seconds after cut off,

$$p = \left[\sqrt{p_3} - \frac{tKA_oC_d}{V_2\sqrt{\frac{\rho}{g}}}\right]^2 \tag{23}$$

where:

- $\mathbf{P_3}$ = pressure in the tubing at pump cut off
- $\mathbf{V_2}$ = total volume of fuel under compression at cut off.

Example: Calculation of injection pressure and duration, considering compressibility but neglecting pressure waves.

Given cam lift per Fig. 54 and the following conditions:

A_0 = 0.000314 sq. in. for .020 in. dia. orifice	C_d = 0.94
A_p = 0.0985 sq. in. for 0.354 in. dia. plunger	L = 34 in.
A_t = 0.0148 sq. in. for 0.138 bore tubing	K = 284,000 psi
P_a = 2850 psi	ρ = 0.0307 lb/cu.in.

The plunger displacement from port closing to top of stroke is (.405 - 0.130) 0.0985 = 0.027 cu. in. Volume of the discharge tubing bore is (34 x 0.148) = 0.509 cu. in. The pump clearance and other small volumes amount to 0.0 16 cu. in. so that the total volume at the beginning of fuel compression is 0.027 + 0.509 + 0.016 = 0.552 cu. in. During injection the maximum displacement of fuel by the plunger from port closing to port opening is (0.370 - 0.130) 0.0985 = 0.024 cu. in. As this is only 4.3 percent of the total volume. V_θ can be assumed constant. Then, taking the cam angle interval θ_a, - θ_b, as 2 degrees, the various constants for Eq. (20) determined from the data are:

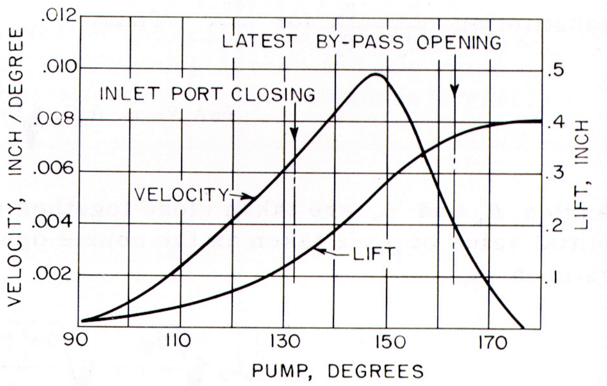


Fig. 54. Plunger lift and velocity curves of NACA pump[4]

$$A_o C_d \sqrt{\frac{2g}{\rho}} = 4.69 \times 10^{-2} ; \quad \frac{2A_o^2 C_d^2 g}{\rho} = 2.20 \times 10^{-3}$$

$$\frac{12n}{\theta_b - \theta_a} \frac{V_\theta}{K} = 0.875 \times 10^{-2} ; \quad \frac{144^2 A_p v_\theta V_\theta}{(\theta_b - \theta_a) K} = \frac{12nV_\theta}{(\theta_b - \theta_a) K} \times 12nA_p v_\theta = 7.76 \, v_\theta$$

$$\frac{144n^2 V_\theta^2}{(\theta_b - \theta_a)^2 K^2} p_a = \left[\frac{12n \; V_\theta}{(\theta_b - \theta_a) K} \right]^2 p_a = 0.765 \times 10^{-4} \, p_a$$

Substituting these values in Eq. (20) and simplifying:

$$\sqrt{p_b} = -5.36 + \sqrt{28.8 + 1.015 \times 10^5 V_\theta + p_a}$$

Beginning with port closing this equation is solved for every two degree intervals. The initial pressure of p_b - p_a is 2850 psi, and thereafter the value of p_a for any 2-deg. interval is the p_a value of p_b from the previous interval. Since the values are computed to the closest 10 psi, values of 5 and 30 are used for 5.36 and 28.8, respectively. The results of the calculated injection pressures up to cut-off are given in Table 2.

The total fuel quantity discharged up to pump cutoff equals the plunger displacement less the fuel absorbed by compression. Therefore, the quantity of fuel injected is,

$$Q = Q_D - \frac{p_2 - p_1}{K} V_2 \tag{24}$$

where:

- $\mathbf{Q_D}$ = effective plunger displacement, 0.024 cu. in.
- $\mathbf{p_2}$ = cut off pressure (from Table 2), 4750 psi
- $\mathbf{p_1}$ = initial pressure (assumed nozzle closing pressure), 2000 psi
- $\mathbf{V_2}$ = volume of compressed liquid (.509 t .016), 0.525 cu. in.

Then, $$Q = 0.024 - \frac{(4750 - 2000)}{284{,}000} 0.525 = 0.019 \text{ cu. in.}$$

With a pump having a non-retracting delivery valve, fuel injection continues after cut- off as the pressure expands from 4750 psi down to the nozzle closing pressure of 2000 psi. The injection pressures after cut -off are obtained by substituting the values in Eq.(23). Accordingly,

$$p = \left[\sqrt{4750} - \frac{284 \times 10^3 \times 0.314 \times 10^{-3} \times 0.94}{0.525 \times \sqrt{2 \times 0.795 \times 10^{-4}}}\right] = (69 - 1.26 \times 10^4 t)^2$$

The solution of this equation for every 2 degree interval is recorded in Table 3, which shows that the injection continues from the nozzle for over 8 degrees after pump cutoff.

MATHEMATICAL METHOD CONSIDERING PRESSURE WAVES[5]

This method is based on the previous equations (9) and (10) that: (1) A change of pressure at any point in the fluid is propagated to all points in the fluid with the speed of sound, $v_s = \sqrt{\frac{Kg}{\rho}}$; and (2) with a change in velocity there is a corresponding change in pressure,

$$\frac{\text{Change of pressure}}{\text{Change of velocity}} = \frac{K}{v_s} = \sqrt{\frac{K\rho}{g}}$$

Thus, as the plunger closes the ports a pressure wave $p_{f1} = v_{f1}\sqrt{\frac{K\rho}{g}}$ is propagated through the tubing and reaches the nozzle in $T = \frac{L}{v_s}$ seconds, where v_{f1} is the fuel velocity in the pump end of the tubing at port closing, and L is length of tubing plus nozzle passages. With variable plunger velocity the forward velocity of the fuel at the pump changes, so that pressure waves of changing amplitude are continually being directed towards the nozzle. For simplicity, L changes in pressure and velocity are considered as taking place in step intervals of $T = \frac{L}{v_s}$.

When the first pressure wave reaches the nozzle, and if the nozzle opening pressure is greater than the wave pressure, the wave will be completely reflected. If the pressure wave p_{f1} is sufficient to open the nozzle, injection begins and the velocity through the tubing relative to the nozzle is,

$$v_t = c_d \frac{A_o}{A_t}\sqrt{\frac{2g\,(P_{f1} - P_c)}{\rho}} \qquad (25)$$

where:

- $\mathbf{P_C}$ = Combustion chamber pressure

Since the nozzle is a restriction in the system, the velocity of flow through the nozzle orifice is generally less than the velocity of flow towards the nozzle so that,

$$v_o = v_f - v_r \qquad (26)$$

where:

- $\mathbf{v_r}$ = velocity reduction after wave reflection

From Eq. (10) a corresponding build up of pressure, $P_r = v_r\sqrt{\frac{K\rho}{g}}$, occurs in the reflected wave traveling towards the pump and reaching there in $\frac{2L}{v_s}$ seconds after the start of the first pressure wave.

When the reflected wave reaches the pump, the pressure in the tubing will increase to $P_f + P_r$ and the forward velocity will momentarily reduce to $v_f - v_r$. As the plunger is still displacing oil at velocity v_f, the forward velocity of the oil will be immediately raised to v_f. Since this results in a forward velocity change of v_r, a corresponding pressure increase occurs raising the pressure to $P_f + 2\,P_r$.

This increased pressure wave then travels towards the nozzle. The increased pressure increases the velocity of discharge through the nozzle, but on account of the nozzle restriction the increase is less than the forward velocity in the tubing. Therefore, another pressure wave is reflected from the nozzle and slows down the flow in the pipe until it reaches the pump. Again the returning pressure wave is completely reflected from the pump and the pressure in the tubing is further increased. Thus, during the effective pump stroke a forward pressure wave is partially reflected at the nozzle end of the tubing, but a returning pressure wave is completely reflected at the pump end.
This process continues until the end of the plunger effective stroke, and the delivery valve closes. For the remainder of the injection process the condition at the pump end is that of a closed pipe. Since the valve is closed, the velocity is zero, and each arriving wave is reflected completely. From Eq. (10)

$$v = \sqrt{\frac{g}{K\rho}}\,(P_f - P_r) = 0;\; P_f = P_r$$

TABLE 2

Computation of Instantaneous Injection Pressures up to Cutoff

1	2	3	4	5	6	7	8	9	10
θ	Pump Deg.	$V_\theta \times 10^3$ in./deg.	$1.015 \times 10^3 V_\theta$	(4) + 30	P_a + (5)	$\sqrt{(6)}$	$\sqrt{P_b}$	P_b Psi	$\frac{t}{\text{sec} \times 10^4}$
0	132	6.6	670	700				2850	0
2	134	6.9	700	730	3580	60	55	3010	4.4
4	136	7.3	740	770	3780	61	56	3140	8.9
6	138	7.8	790	820	3960	63	58	3360	13.3
8	140	8.4	850	880	4240	65	60	3600	17.8
10	142	8.9	900	930	4530	67	62	3840	22.2
12	144	9.4	950	980	4820	69	64	4100	26.6
14	146	9.8	990	1020	5120	72	67	4500	31.1
16	148	9.8	990	1020	5520	74	69	4750	35.5
18	150	9.5	960	990	5740	76	71	5050	40.0
20	152	8.9	900	930	5980	77	72	5200	41.4
22	154	8.2	830	860	6060	78	73	5350	48.8
24	156	7.2	730	760	6110	78	73	5350	53.3
26	158	6.3	640	670	6020	78	73	5350	57.7
28	160	5.4	550	580	5930	77	72	5200	62.2
30	162	4.6	470	500	5700	76	71	5050	66.6
32	164	3.8	390	420	5470	74	69	4750	71.1

TABLE 3

Computations of Instantaneous Injection Pressures after Cutoff

1	2	3	4	5	6
Sec $\times 10^4$	Deg.	Total Deg.	$1.26 \times 10^4 t$	69 - (4)	p psi
0	0	164	0	-	4,750
4.4	2	166	6	63	3,900
8.9	4	168	11	58	3,400
13.3	6	170	17	52	2,700
17.8	8	172	22	47	2,200
22.2	10	174	28	41	1,700

The pressure P_x existing at any point in the discharge tubing at a certain time can be expressed as,

$$P_x = P_r \pm P_f + P_R \tag{26}$$

where:

- $\mathbf{P_R}$ = residual pressure in the tubing
- $\mathbf{P_f}$ = sum of forward pressure waves from pump towards nozzle
- $\mathbf{P_r}$ = sum of waves returning towards pump up to that time

The velocity coexisting with the pressure is related to the pressure as given by Eq, (10) Then from Eq. (26) the resultant velocity is,

$$v_x = v_f - v_r = (P_f - P_r)\sqrt{\frac{g}{K}} \tag{28}$$

<u>Example.</u> Calculation of pressures and fuel velocities at the pump and nozzle end of the discharge tubing.

Given constant velocity cam per Fig. 55 and the following conditions:

- $\mathbf{A_0}$ = 0,00076 sq. in. for 7 – 0,30 mm orifices
- $\mathbf{A_P}$ = 0,0986 sq. in. for 9 mm dia. plunger
- $\mathbf{A_t}$ = 0,0085 sq. in. for .104 in. bore tubing
- $\mathbf{P_a}$ = 2500 psi
- $\mathbf{P_d}$ = 1815 psi (closing pressure)
- $\mathbf{P_c}$ = 400 psi
- $\mathbf{P_R}$ = 400 psi
- $\mathbf{C_d}$ = 0,65
- **L** = 18 in.
- **n** = 1000 rpm
- **k** = 260,000 psi
- **ρ** = 0,0308 lb./cu.in.

The accoustic velocity of the oil in the tubing is from Eq. (9):

$$v_s = \sqrt{\frac{260{,}000 \times 386}{0.0308}} = 57{,}200 \text{ ips or } 4766 \text{ fps}$$

In this analysis the injection period will be divided into separate phases of time, T, required by the pressure waves to travel from the pump to the nozzle or from the nozzle to the pump. Velocities and pressures during each phase will be calculated and plotted as pressure-time and velocity diagrams for the conditions occurring during each phase at both the pump and nozzle ends of the discharge tubing.

The time for a pressure wave to travel from one end of the tubing to the other is,

$$T = \frac{L}{v_s} = \frac{18}{57{,}200} = 0.000315 \text{ sec.}$$

From Fig. 55 the effective pump stroke lasts 8 cam degrees,

and at 1000 rpm this is $\dfrac{8 \times 60}{360 \times 1000} = 0.00133$ sec.

During this time there will be $\dfrac{0.00133}{0.000315} = 4.22$ traverses of pressure waves from one end of the tubing to the other.

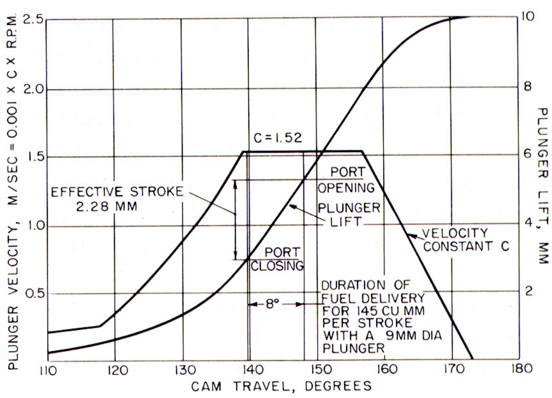


Fig. 55. Plunger lift and velocity curves.

According to Fig. 55 the plunger velocity during injection is constant and equals

$$\frac{0.001 \times 1.52 \times 1000 \times 39.37}{12} = 5 \text{ fps.}$$

Then the forward velocity of the fuel entering the tubing at the pump end is,

$$v_f = \frac{A_p}{A_t} \times v_p = \frac{0.0986}{0.0085} \times 5 = 58 \text{ fps.}$$

Phase 1

During this interval of time T the pressure wave travels from the pump to the nozzle. At the pump end the fuel velocity is 58 fps. From Eq. (10) the pressure change in increasing the fuel velocity in the tubing from 0 to 58 fps is,

$$\sqrt{\frac{0.0308 \times 260{,}000}{386}} = 3170 \text{ psi.}$$

As the residual line pressure is 400 psi, the total pressure at the pump is 3170 + 400 = 3570 psi.

The pressure wave at 3570 psi travels towards the nozzle and efflux begins, since the nozzle valve opening pressure is only 2500 psi. Velocity of flow through the tubing in relation to the nozzle from Eq. (25) is,

$$\frac{0.65 \times .00076}{0.0085 \times 12}\sqrt{\frac{772(3570 - 400)}{0.0308}} = 43.2 \text{ fps.}$$

The velocity of the fuel first arriving at the nozzle was 58 fps, so it is apparent that the fuel flow is restricted. The forward pressure wave is, therefore, partially reflected, and its amplitude P_r is proportional to the reduction in the forward fuel velocity.

The actual rate of fuel discharge depends upon the readjusted pressure difference across the nozzle orifices. From Eq. (27) the total fuel pressure is, $P_x = P_r + P_f + P_R$, and so the actual velocity through the orifice is

$$v_o = \sqrt{\frac{2g}{\rho}(p_r + p_f + p_R - p_o)}.$$

The equivalent velocity in the tubing is, $v_t = \frac{A_o}{A_t} C_d \sqrt{\frac{2g}{\rho}(p_r + p_f + p_R - p_o)}$. From Eq. (28)

$$v_f - v_r = (p_f - p_r)\sqrt{\frac{g}{\rho K}}$$

and since $v_f - v_r = v_t$

$$C_d \frac{A_o}{A_t}\sqrt{\frac{2g}{\rho}(p_r + p_f + p_R - p_c)} = (p_f - p_r)\sqrt{\frac{g}{\rho K}}$$

Dividing both sides of this equation by $\sqrt{\frac{\rho g}{K}}$ it becomes:

$$C_d \frac{A_o}{A_t}\sqrt{2K(p_r + p_f + p_R - p_c)} = p_f - p_r \quad \text{and} \quad p_r + C_d \frac{A_o}{A_t}\sqrt{2K(p_r + p_f + p_R - p_c)} = p_f$$

Substituting numerical values in the equation,

$$p_{r1} + 0.65 \times \frac{.00076}{.0085}\sqrt{2 \times 260{,}000\,(p_{r1} + p_f + 400 - 400)} = P_f$$

$$p_{r1} + 42\sqrt{p_{r1} + p_f} = p_f \qquad (29)$$

Thus, P_f under the radical in this last equation represents the total effective pressure (P_x -p_c) at the nozzle orifice before reflection of the approaching pressure wave takes place. Since P_f = 3170 psi, P_{rl} can be determined:

$$P_{r1} + 42\sqrt{3170 + P_{r1}} = 3170; \quad P_{r1} = 590 \text{ psi.}$$

The total pressure at the nozzle at the end of the first phase is P_{xl} 3570 + 590 = 4160 psi. The velocity in the tubing corresponding to that through the orifice with this total pressure is,

$$v_t = \frac{A_o}{A_t}\frac{C_d}{12}\sqrt{\frac{2g}{\rho}}\,(p_{x1} - p_c) = \frac{0.00076}{0.0086} \times 0.65\sqrt{\frac{2 \times 386}{0.0308}}\,(4160 - 400) = 46.5 \text{ fps}$$

Phase 2

During this interval the pressure wave returns to the pump end of the tubing and is completely reflected by the advancing plunger. At the nozzle end the pressure is 4160 psi and the velocity 46.5 fps as determined for Phase 1. At the pump end the fuel velocity from the plunger is still 58 fps. The total pressure at the pump end is,

$$P_{x2} = P_{x1} + P_{r1} = P_f + P_R + P_{r1} + P_{r1} = 4160 + 590 = 4750 \text{ psi.}$$

Phase 3

The re-reflected wave travels to the nozzle during the interval. When it reaches the nozzle orifices, it is partially reflected because of velocity reduction from the orifice restriction. Fuel velocity and pressure at the pump end continue at 58 fps and 4750 psi, respectively.

Conditions at the nozzle end are obtained using Eq. (29),

$$P_{r3} + 42\sqrt{P_{x2} + P_{r3} - P_c} = P_f; \quad P_{r3} + 42\sqrt{4350 + P_{r3}} = 3170; \quad P_{r3} = 305$$

Total pressure is $P_{x3} = P_{x2} + P_{r3}$ = 4750 + 305 = 5055 psi.

Velocity in tubing corresponding to that through the nozzle orifice is,

$$v_{t3} = \frac{0.00076}{0.0086} \times 0.65 \times 13.2\sqrt{(5055 - 400)} = 52.3 \text{ fps.}$$

Phase 4

During this interval the wave travels towards the pump again with the fuel velocity and pressure at the nozzle remaining at 52.3 fps and 5055 psi, respectively. The 305 psi reflected wave travels back to the pump and is reflected raising the pressure to $P_f + 2P_r$ =4750 + 2 x 305 = 5360 psi. As the plunger is still delivering fuel, its velocity continues at 58 fps.

Phase 5

The plunger ceases fuel delivery to the tubing 0.22 x .000315 = 6.9 x 10^{-5} seconds after phase 4 when the pressure wave has travelled 22 percent of the distance to the a nozzle. At this time a negative pressure wave of 3170 psi, equivalent to the change of fuel velocity from 58 fps to 0, starts off towards the nozzle as shown in Fig. 56a. The pressure wave, represented by the shaded block, travels on to the nozzle and is partially reflected (160 psi). The negative pressure wave travelling towards the nozzle arrives 6.9 x 10^{-5} seconds later and reduces the total pressure by 3170 psi. Thus, the nozzle experiences a 305 + 160 = 465 psi pressure peak that lasts only 6.9 x 10^{-5} seconds, and the total pressure for that time is 5520 psi.

The total pressure before the wave is reflected is,

$$P_{x5.22} = 5055 + 465 - 3170 = 2350 \text{ psi}$$

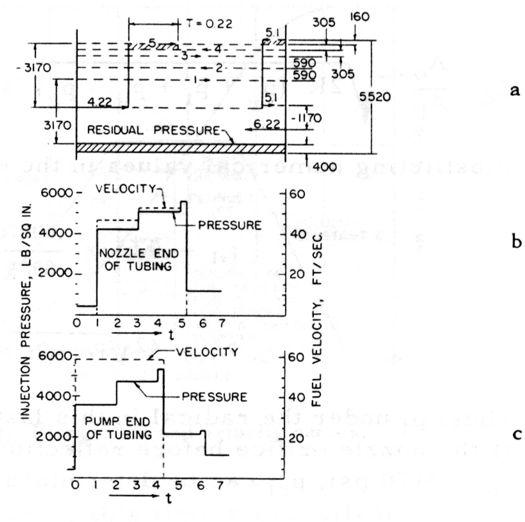


Fig. 56. Velocity and pressure-time diagrams.

The value of the reflected pressure wave is,

$$p_{r6} + 42\sqrt{p_{x5.22} + p_{r6} - p_c} = p_f$$

$$p_{r6} + 42\sqrt{1950 + p_{r6}} = p_f = 0; \; p_{r6} = -1170 \text{ psi}$$

This reduces the total pressure at the nozzle end of the tubing to

$P_{x5.22} = 2350 - 1170 = 1180$ psi

As this is below the nozzle valve closing pressure of 1875 psi, injection terminates.

From the values obtained for the various phases, pressure-time and velocity-time diagrams for the injection interval can be constructed as shown in Fig. 56b and c.

GRAPHICAL METHOD CONSIDERING PRESSURE WAVES[6]

This method of analysis involves construction of diagrams of t - x, or time-distance, and v - p, or velocity-pressure. The velocity of propagation, vs' of a pressure wave can be graphically represented in the t - x diagram by lines of constant slope, Fig. 57, since,

$$\tan(\pm\phi) = \frac{\Delta L}{\Delta T} = \frac{L}{\frac{L}{v_s}} = \pm v_s \tag{30}$$

Thus, a pressure wave initiated at the pump travels towards the nozzle with a velocity v_s, and the locus of the wave front will lie in a straight line of slope tan φ. Similarly, a disturbance from the nozzle end of the discharge tubing will be characterized by a slope of tan (-φ). As before, the traverse time through the tubing is $T = \frac{L}{v_s}$.

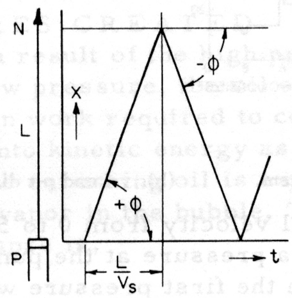


Fig. 57. Time-distance (t-x) diagram.

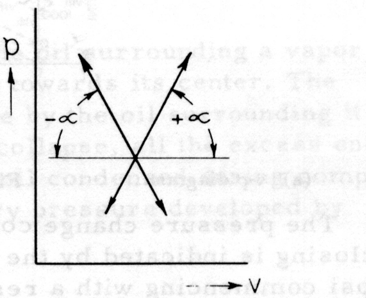


Fig. 58. Velocity-pressure (v-p) diagram.

The proportionality of velocity change to pressure change is represented in the v -p diagram by lines of constant slope, Fig. 58, from

$$\frac{\Delta p}{\Delta v} = \pm\frac{K}{v_s} = \tan(\pm\alpha) \tag{31}$$

In the v - p diagram the pump, tubing, and nozzle can be represented by characteristic lines. The intersection of these in suitable manner, determines the pressure and velocity at a time t or distance x. The character of the entire injection process depends largely on the rate of fuel discharge from the nozzle. since this determines the velocity of the fuel in the nozzle end of the tubing. It is expressed by the parabola,

$$V_t = \frac{A_o}{A_t} C_d \sqrt{\frac{2g(p_n - p_c)}{\rho}} \tag{32}$$

where:

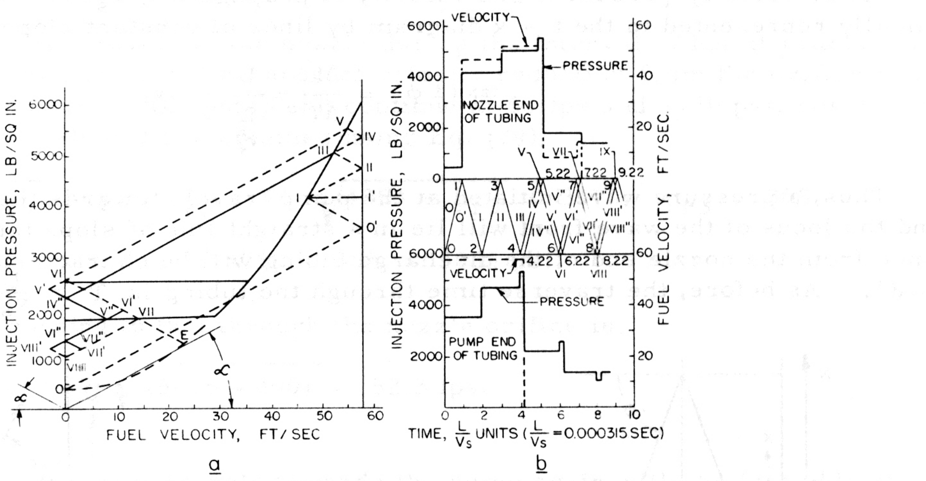
- $\mathbf{P_n}$ = pressure in tubing at nozzle end, psi

Example. Determine the graphical solution for the simple injection system and conditions of the previous example solved mathematically.

From Eq. 31, $\tan \pm \alpha = \frac{K}{v_s} = \frac{260{,}000}{4766} = 54.5 \text{ psi/fps}^2.$ This determines the slope of the pressure wave reflections in the v - p diagram; and it reveals that for each foot per second velocity change, there is a pressure change of 54.5 fps.

The parabola is constructed in the p - v diagram of Fig. 59 by substituting various values of injection pressure in Eq. 32 to get corresponding values of v_t.

It represents the fuel velocity in the nozzle end of the tubing for efflux of fuel through the nozzle at various pressures. The velocity of the fuel entering the tubing at the pump end is indicated by the vertical full line at the right. while the zig-zag lines represent the course of pressure waves between pump and nozzle.



(a) v-p diagram. **Fig. 59. Graphical analysis of jerk pump system.** (b) v-t and p-t diagrams.

The pressure change corresponding to the change in fuel velocity from 0 to 58 fps at port closing is indicated by the line 00' of slope α. This gives a pressure at the pump (0') of 3550 psi commencing with a residual pressure of 400 psi. When the first pressure wave arrives at the nozzle. point I on the parabola. it is partially reflected since the velocity through the nozzle orifices is less than that of the pressure wave. This results in the pressure raising to 4150 psi to complete phase 1. The partially reflected wave travels back to the pump. and it is completely reflected as represented by point H. Complete reflection is indicated by 0'I' = I'II. In this

manner points I to V are determined. In Fig. 59 the duration of injection at the pump and nozzle is represented in time units of $\frac{L}{v_s}$. Thus, the lines 0-1, 1-2, etc. show the position of the pressure wave in the tubing at any time during injection. At T = O.

the pressure in the tubing at the pump end is instantaneously raised to 3550 psi while at the nozzle the residual pressure is 400 psi. The pressure at the pump remains at 3550 psi until the wave reflected from the nozzle arrives back in time. The pressure then rises abruptly to 4700 psi and remains there up to $\frac{4L}{v_s}$ The action of the pressure waves can be traced in this manner up to point IV.

As the pump delivery lasts 4.22 $\frac{L}{v_s}$ seconds. the pressure waves propagated at the L Vs pump from 4 to 4.22 are restored to a velocity of 58 fps and travel to the nozzle as represented by point V. At 4.22 $\frac{L}{v_s}$ however. the velocity of the fuel at the pump is reduced to zero and the pressure at the pump is immediately reduced to 2200 psi (point IV"). The waves represented by point V arrive at the nozzle 0.22 $\frac{L}{v_s}$ seconds earlier than those represented by Vs V". The waves are then reflected to V1. V11, V111, and to V1, V11, and V111, respectively.

When the pressure wave V1 at the pump arrives at the nozzle, it would normally reduce the pressure to E and the nozzle valve would close. As the nozzle starts to close, the controlling efflux area is transferred from the orifice to the valve seat, and the pressure and velocity of the fuel at the nozzle lie on the closing pressure line V11.

The fuel pressures and velocities at the pump and nozzle end of the tubing are plotted for the duration of injection in Fig. 59. These results agree closely with the mathematical method up to 5.22 L/vs. The nozzle valve closes at that point according to the mathematical method, whereas it does not close until 7.22 L/vs by the graphical method. With more attention to accuracy in the mathematical method, the results of both would agree more closely.

These methods of analysis depict the effect that various changes in components and conditions have on the injection characteristics. Therefore, they may be helpful for preliminary analysis and in reducing testing time.

CAVITATION EROSION

Pressure waves not only alter the injection characteristics of a fuel system, but they may cause premature failure of some of the components by cavitation erosion. The cavitation process consists in the formation and collapse of vapor bubbles in a flowing liquid. These bubbles form at any point in the fuel system where the pressure drops to or below the vapor pressure of the fuel at that temperature, and they collapse when exposed to a pressure higher than the vapor pressure. Very high pressures are created by the collapse of these bubbles, and under cavitation conditions the constant bombardment of the surface at these pressures erodes the exposed metal.

PRESSURES CREATED

As a result of the high pressure and surface tension of the oil surrounding a vapor bubble at low pressure, the oil outside the bubble is accelerated towards its center. The difference in work required to compress the bubble and that done by the oil surrounding it is converted into kinetic energy as the oil rushes in. On complete collapse, all the excess energy stored in the onrushing oil is expended on the minute amount of oil condensed during compression of the vapor in the bubble. The equation[7] for this momentary pressure developed by bubble collapse is,

$$p_\sigma = \sqrt{K\,(p_f - p_v)\left(\frac{V_s}{V_w}\right)^{1/3}} \qquad (33)$$

where:

- **P_f** = pressure of oil
- **P_V** = vapor pressure in bubble
- **V_S** = specific volume of oil vapor in bubble
- **V_W** = specific volume of condensed oil.

Example. Assume surge pressure of 1500 psi and oil temperature of 150 d eg, F. From Fig. 53, K is 225,000 psi. According to the gas equation, pV = MRT, the volume of one pound of no. 1 diesel fuel vapor at 150 deg. F. and an assumed absolute pressure of 1 psi is,

$$V = \frac{MRT}{p} = \frac{1 \times 1545 \times (460 + 150)}{180 \times 1 \times 144} = 36.3 \text{ cu. ft.}$$

For no. 1 diesel fuel V_W= 0.018 cu.ft./lb., so from Eq. 33

$$p_\sigma = \sqrt{225{,}000 \times 1500 \times \left(\frac{36.3}{0.018}\right)^{1/3}} = 96{,}500 \text{ psi}$$

EFFECTS

Evidence indicates that cavitation erosion is a combination of physical and chemical destruction. The predominant factor, however, is physical, and the erosion generally starts at a crevice or fissure resulting from either design or flaw in the material. It may be found in any of the components of the injection system in contact with the fuel.

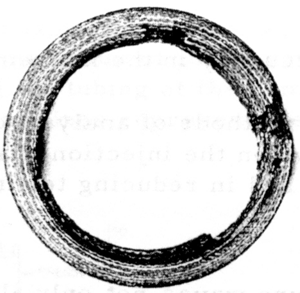


Fig. 60. Delivery valve gasket erosion.

Delivery Valve Gasket

Serious erosion is frequently experienced with the conventional copper delivery valve gaskets (Fig. 60). The condition is sometimes alleviated by using softer copper of 40 Rockwell F maximum, which flows more on tightening so that no crevices are produced with the mating parts. The most effective remedy is to eliminate the gasket entirely, which is done in pumps of latest design.

Baffle Screws

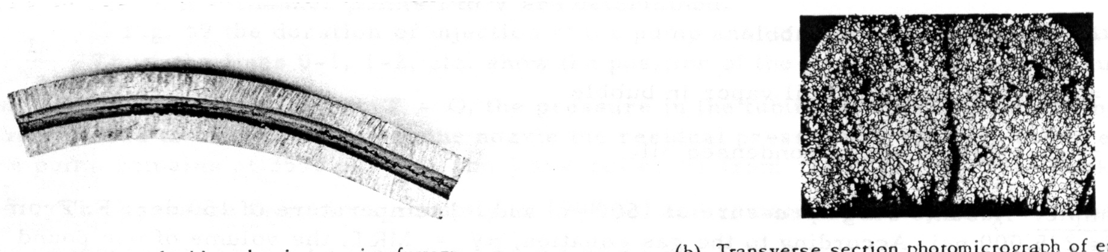
These screws are subjected to the impingement of fuel spilled through the barrel port as it is uncovered by the plunger helix. Rapid erosion of soft steel or thin case-hardened screws has been experienced. This has been corrected by screws with hardened tool steel tips or by using a hardened steel baffle ring similar to that of the General Motors unit injector.

Plunger and Barrel.

Erosion of the metering portion of the plunger sometimes extends over an area larger than the metering port diameter, so that it apparently occurs over a wide load range. It may also occur in the barrel with the formation of several cavities around the port. Where the condition is serious some relief has been provided by providing small interconnecting ports on opposite sides of the plunger metering land.

Discharge Tubing

A typical example of cavitation in tubings is shown in Fig. 6la. It manifest itself in long, straight, furrow-like cavities. Surface quality of the tubing bore is an important factor as indications are that crevices in the tubing act as focal points from which cavitation erosion starts. Fig. 61 b shows a photomicrograph taken at 100X of a cross section from an eroded tubing. In this case the cavitation erosion extends from the bore to the outer surface of the tubing.



(a) Section of tubing showing erosion furrow. (b) Transverse section photomicrograph of eroded tubing.

Fig. 61. Cavitation erosion in discharge tubing.

Nozzle and Holders

Erosion of fuel passages in nozzle holders has been experienced similar to that in the discharge tubing. In nozzles cavitation erosion sometimes occurs both on the valve and body at the seat and just above the fuel sump where there is a slight clearance between the valve and the body bore. In this case a design type of "crevice" localizes the cavitation damage.

CAVITATION CONDITIONS

Pressure oscillograms provide a means for determining the presence of cavitation. Under certain conditions the pressure in the injection system drops to the fuel vapor pressure, and the normal negative wave portions of the oscillogram appear as straight horizontal lines instead of a sinusoidal shape. As a consequence of the inability of the fuel to withstand tension or drop below zero absolute pressure, vapor bubbles form. As shown in Fig. 62, (see following page) high speed photographs of haze in a transparent section of tubing during the flat intervals of the pressure traces following the main injection revealed the formation of bubbles at these times[8]. Cavitation in this instance is the result of the delivery valve retraction volume being more than the expansion volume of the fuel in the tubing, so that a portion of the tubing is filled with dissolved air and vapor escaping from the fuel.

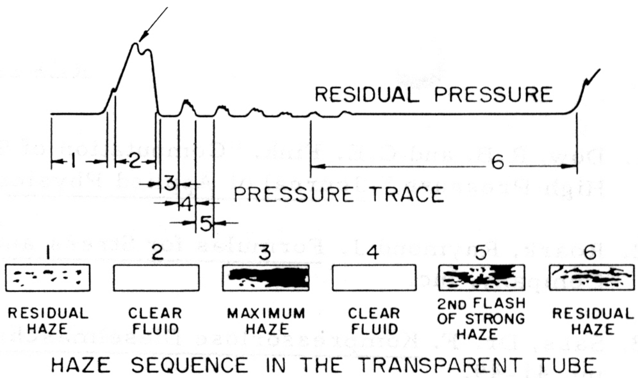
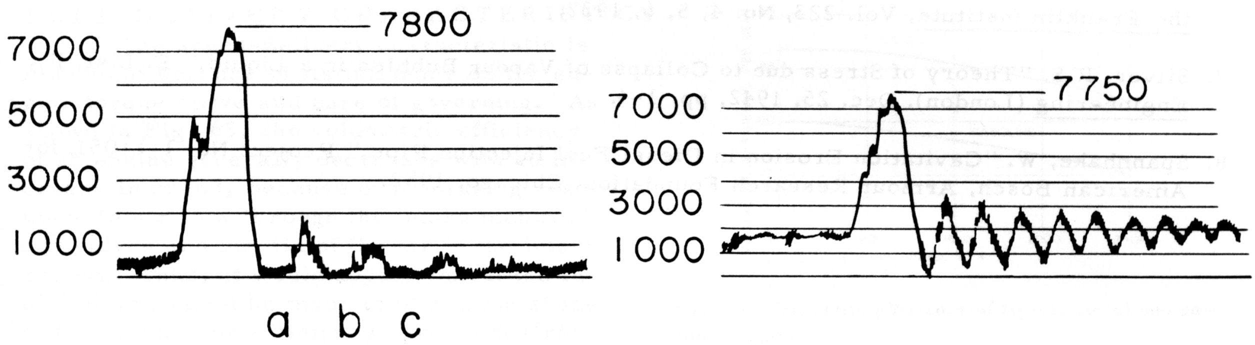


Fig. 62. Correlation of pressure trace and appearance of fuel in transparent tube.

CORRECTION.

An apparent method of eliminating cavitation is to reduce the retraction volume of the delivery valve so that the residual pressure never goes low enough to allow bubbles to form. This is shown in Fig. 63 where changing the delivery valve retraction from 800 cu, down to 400 c u. increased the residual pressure as high as 1500 psi and eliminated the flat portions of the pressure trace after the main injection.



800 cu. mm retraction

400 cu. mm retraction

Fig. 63. Effect of delivery valve retraction on pressure trace. 20mm plunger, 3600 psi nozzle o.p., tubing .125 × 63 in., 370 rpm, 1700 cu. mm per stroke

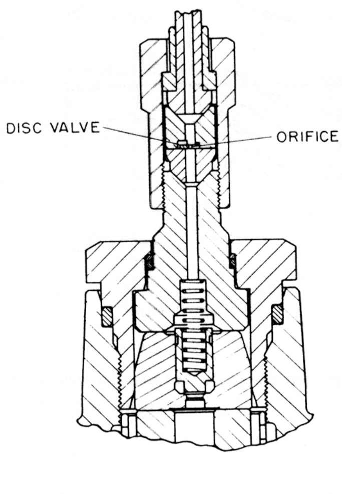


Fig. 64. Snubber valve at pump outlet.

Simply reducing the delivery valve retraction may not be the solution, because the original retraction volume may be required to eliminate secondary injections. In such cases cavitation can be eliminated by adding a snubber valve at the discharge outlet. As shown in Fig. 64 this consists of a small disc valve which permits unrestricted flow during fuel delivery from the pump, but after spill the disc valve seats and its small central hole allows the pressure in the tubing to bleed down at a rate to give sharp cut-off with adequate residual pressure.

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